WHAT IS A SET?

A set is any well-defined list or collection of things, and will be denoted by capital letters A, B, X, Y,…….

Below you'll see just a sampling of items that could be considered as sets:

- Your favorite clothes
- A coin collection
- The items in a store
- The English alphabet
- Even numbers

A set could have as many entries as you would like.
It could have one entry, 10 entries, 15 entries, infinite number of entries, or even have no entries at all!

For example, in the above list the English alphabet would have 26 entries, while the set of even numbers would have an infinite number of entries.

Each entry in a set is known as an **element or member** and will be denoted by lower case letters a,b,x,y,.....

Sets are written using curly brackets "{" and "}", with their elements listed in between.

For example the English alphabet could be written as

{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u,v,w,x,y,z}

and even numbers could be {0,2,4,6,8,10,...} (Note: the dots at the end indicating that the set goes on infinitely)

Elements

By now you know each entry in a set is called an **element**

Principles:

- $\in$ belong to
- $\notin$ not belong to
- $\subseteq$ subset
- $\subset$ proper subset
- $\not\subset$ not subset

$\in$ means "belong to"; $\notin$ means "not belong to"

So we could replace the statement "a is belong to the alphabet" with a $\in$ {alphabet} and replace the statement "3 is not belong to the set of even numbers" with 3 $\notin$ {Even numbers}

Now if we named our sets we could go even further.
Give the set consisting of the **alphabet** the name A, and give the set consisting of **even numbers** the name E.

We could now write

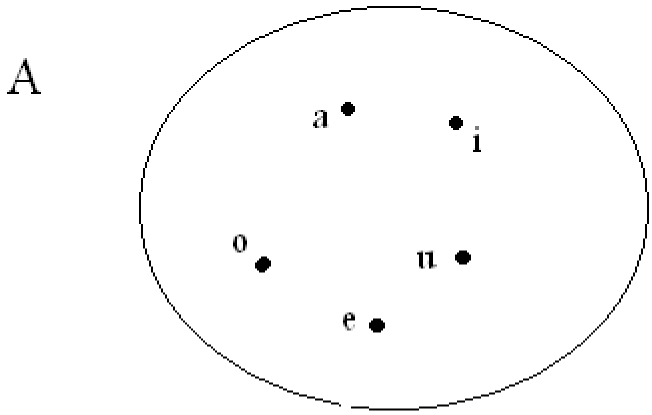
$a \in A$

and

$3 \notin E$.

There are three ways to specify a particular set:

1) By list its members (if it is possible), for example, A= {a,e,i,o,u}
2) By state those properties which characterize the elements in the set, for example, A={x:x is a letter in the English alphabet, x is a vowel}
3) Venn diagram : (A graphical representation of sets).



Example (1)

A={x:x is a letter in the English alphabet, x is a vowel}

e $\in$ A (e is belong to A)
f $\notin$ A (f is not belong to A)

Example (2)

X is the set {1,3,5,7,9}

3 $\in$ X
4 $\notin$ X

Universal set, empty set:

In any application of the theory of sets, the members of all sets under investigation usually belong to some fixed large set called the universal set. For example, in human population studies the universal set consists of all the people in the world. We will let the symbol U denotes the universal set.
The set with no elements is called the empty set or null set and is denoted by $\varnothing$ or {}.

Subsets:

Every element in a set A is also an element of a set B, then A is called a subset of B. We also say that B contains A. This relationship is written:

$A \subset B$ or $B \supset A$

If A is not a subset of B, i.e. if at least one element of A dose not belong to B, we write $A \not\subset B$.

Example:

Consider the sets.

$A = \{1,3,4,5,8,9\}$ $B = \{1,2,3,5,7\}$ and $C = \{1,5\}$

Then $C \subset A$ and $C \subset B$ since 1 and 5, the element of C, are also members of A and B.

But $B \not\subset A$ since some of its elements, e.g. 2 and 7, do not belong to A. Furthermore, since the elements of A,B and C must also belong to the universal set U, we have that U must at least the set {1,2,3,4,5,7,8,9}.

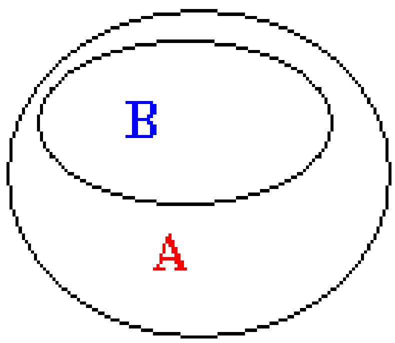
$A \subset B$: $\{\forall x \in A$ $\Rightarrow$ $x \in B$

$A \not\subset B$: $\{\exists x \in A$ but $x \notin B$

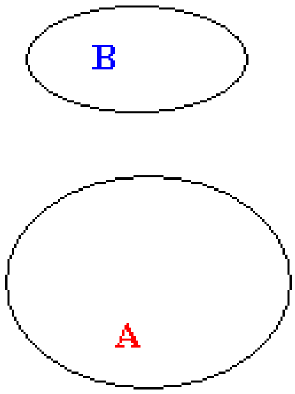
$\forall$: For all لكل

$\exists$: There exists يوجد على الاقل

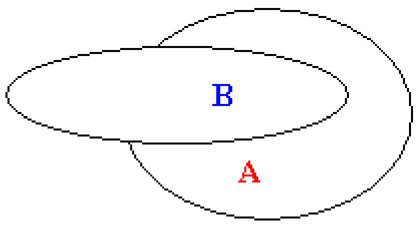
The notion of subsets is graphically illustrated below



In this first illustration B is entirely within A so B ⊂ A.



In this second illustration A and B have nothing in common (A ∩ B = ∅) so we could write
A ⊄ B and B ⊄ A.



In this last illustration some of B is in A, but not all of B is in A so we could write B ⊄ A.

Set of numbers:

Several sets are used so often, they are given special symbols.

The natural numbers

$$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$$

The integers

$$\mathbb{Z} = \mathbb{N} \cup \{\ldots, -2, -1\}$$

The rational numbers

$$\mathbb{Q} = \mathbb{Z} \cup \{\ldots, -1/3, -1/2, 1/2, 1/3, \ldots, 2/3, 2/5, \ldots\}$$

Where $Q = \{ a/b : a, b \in Z, b \neq 0\}$

The real numbers

$$\mathbb{R} = \mathbb{Q} \cup \{\ldots, -\pi, -\sqrt{2}, \sqrt{2}, \pi, \ldots\}$$

The **complex** numbers

$$\mathbb{C} = \mathbb{R} \cup \{i, 1+i, 1-i, \sqrt{2}+\pi i, \ldots\}$$

Where $C = \{ x + iy ; x, y \in R; i = \sqrt{-1}\}$

Theorem 1:

For any set A, B, C:

1- $\varnothing \subset A \subset U$.

2- $A \subset A$.

3- If $A \subset B$ and $B \subset C$, then $A \subset C$.

4- $A = B$ if and only if $A \subset B$ and $B \subset A$.

Set operations:

1) UNION:

A union of two or more sets is another set that contains everything contained in the previous sets.

If A and B are sets then $A \cup B$ represents the union of A and B:

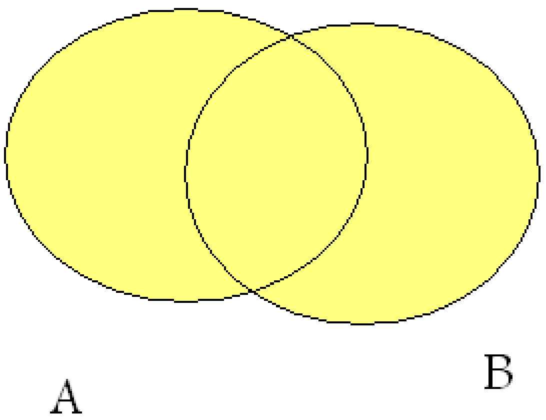
$A \cup B = \{ x : x \in A \text{ or } x \in B\}$

Example

$A=\{1,2,3,4,5\}$ $B=\{5,7,9,11,13\}$

$A \cup B = \{1,2,3,4,5,7,9,11,13\}$

Notice that when I wrote out the united set I did not write "5" twice. I simply listed all of the new sets elements.

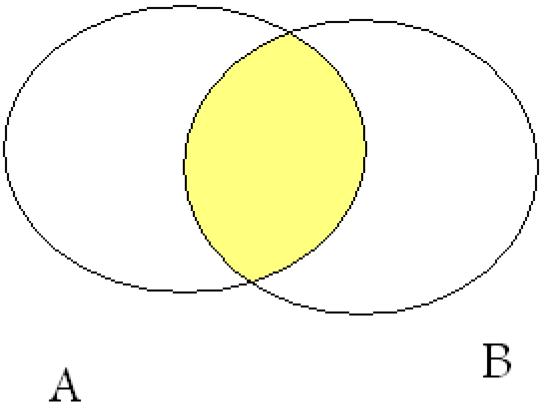


2) **INTERSECTION**

The intersection of two (or more) sets is those elements that they have in common.

So if A and B are sets then the intersection (the elements they both have in common) is denoted by $A \cap B$.

$A \cap B = \{ x : x \in A \text{ and } x \in B\}$.



Example 1:

A={1,3,5,7,9}   B={2,3,4,5,6}
The elements they have in common are 3 and 5

$A \cap B = \{3,5\}$

Example 2

A={The English alphabet}   B={vowels}

So $A \cap B$ = {vowels}

Example 3

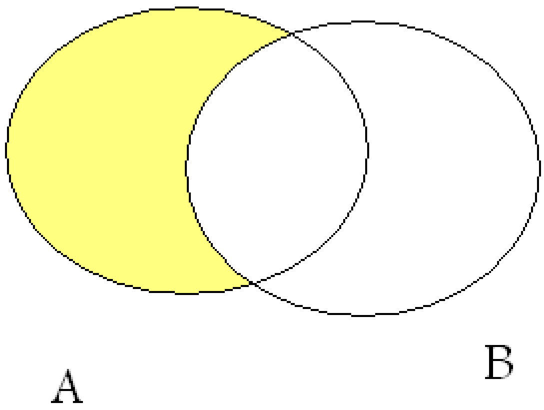
A={1,2,3,4,5}   B={6,7,8,9,10}
In this case A and B have nothing in common.

$A \cap B = \varnothing$

3) THE DIFFERENCE:

The difference of two sets A\B or A-B is those elements which belong to A but which do not belong to B.

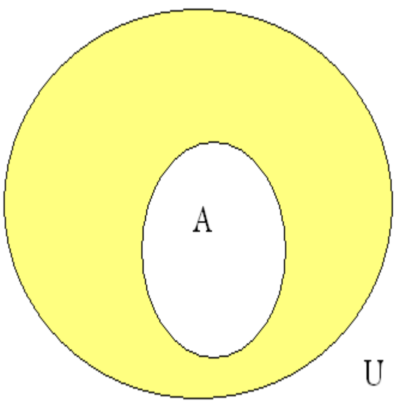
$$A\backslash B = \{x : x \in A, x \notin B\}$$



4) COMPLEMENT OF SET:

Complement of set A^c or A' , is the set of elements which belong to U but which do not belong to A .

$$A^c = \{x : x \in U, x \notin A\}$$



Example :

let $A=\{1,2,3\}$ $B=\{3,4\}$ $U=\{1,2,3,4,5,6\}$

Find :

$A \cup B = \{1, 2, 3, 4\}$

$A \cap B = \{3\}$

$A - B = \{1, 2\}$

$A^c = \{4, 5, 6\}$

Theorem 2 :

$A \subset B$, $A \cap B = A$, $A \cup B = B$ are equivalent

Theorem 3: (Algebra of sets)

Sets under the above operations satisfy various laws or identities which are listed below:

1- $A \cup A = A$
$A \cap A = A$

2- $(A \cup B) \cup C = A \cup (B \cup C)$ Associative laws
$(A \cap B) \cap C = A \cap (B \cap C)$

3- $A \cup B = B \cup A$ Commutatively
$A \cap B = B \cap A$

4- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ Distributive laws
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

5- $A \cup \varnothing = A$ Identity laws
$A \cap U = A$

6- $A \cup U = U$ Identity laws
$A \cap \varnothing = \varnothing$

7- $(A^c)^c = A$ Double complements

8- $A \cup A^c = U$ Complement intersections

$A \cap A^c = \varnothing$ and unions

9- $U^c = \varnothing$

$\varnothing^c = U$

10- $(A \cup B)^c = A^c \cap B^c$ De Morgan's laws

$(A \cap B)^c = A^c \cup B^c$

We discuss two methods of proving equations involving set operations. The first is to break down what it means for an object x to be an element of each side, and the second is to use Venn diagrams.
For example, consider the first of De Morgan's laws :

$$(A \cup B)^c = A^c \cap B^c$$

We must prove: 1) $(A \cup B)^c \subset A^c \cap B^c$

2) $A^c \cap B^c \subset (A \cup B)^c$

We first show that $(A \cup B)^c \subset A^c \cap B^c$

Let's pick an element at random $x \in (A \cup B)^c$. We don't know anything about x, it could be a number, a function, or indeed an elephant. All we do know about x, is that

$x \in (A \cup B)^c$, so

$x \notin A \cup B$

because that's what complement means. Therefore

$x \notin A$ and $x \notin B$,

by pulling apart the union. Applying complements again we get

$x \in A^c$ and $x \in B^c$

Finally, if something is in 2 sets, it must be in their intersection, so

$x \in A^c \cap B^c$

So, any element we pick at random from : $(A \cup B)^c$ is definitely in,

$A^c \cap B^c$, so by definition

$(A \cup B)^c \subset A^c \cap B^c$

Next we show that $(A^c \cap B^c) \subset (A \cup B)^c$.

This follows a very similar way. Firstly, we pick an element at random from the first set, $x \in (A^c \cap B^c)$

Using what we know about intersections, that means

$x \in A^c$ and $x \in B^c$

Now, using what we know about complements,

$x \notin A$ and $x \notin B$.

If something is in neither A nor B, it can't be in their union, so

$x \notin A \cup B$,

And finally

$\therefore \quad x \in (A \cup B)^c$

We have prove that every element of $(A \cup B)^c$ belongs to $A^c \cap B^c$ and that every element of $A^c \cap B^c$ belongs to $A \cup B)^c$. Together, these inclusions prove that the sets have the same elements, i.e. that $(A \cup B)^c = A^c \cap B^c$

EXERCISE:

1- LET A={1,2,4,a,b,c}. Answer each of the following as True or False.

a. $2 \in A$
b. $3 \in A$
c. $c \notin A$
d. $\varnothing \in A$
e. $\varnothing \notin A$
f. $A \in A$

2- Let A={a,b,c,g}
B={d, e,f,g}
C={a,c,f}
D={f,h,k}
U={a,b,c,d,e,f,g,h,k}

Compute: $A \cup B$, $B \cup C$, $A \cap B$, $A \cap C$, $A - B$, $B \cap D$, A^c.

Power set

The *power set* of some set S, denoted P(S), is the set of *all* subsets of S (including S itself). (The empty set is a subset of **all** sets.)

For example, P({0,1})={{},{0},{1},{0,1}}

Example : Let A = { 1,2 3}

Power set of set A = P(A) = [{1},{2},{3},{1,2},{1,3},{2,3},{},A]

Cardinality

The *cardinality* of a set S, denoted |S|, is simply the number of elements a set has. So |{a,b,c,d}|=4, and so on. The cardinality of a set need not be finite: some sets have infinite cardinality.

The cardinality of the power set

If P(S)=T, then $|T|=2^{|S|}$.

This is because T contains sets representing all possible combinations of existence or nonexistence of the elements of P, meaning that each element can be in two states: in the subset, or not in the subset.

Since the number of possible combinations of different states of objects is the multiple of all the number of possible states of each object, and since each element in P can have exactly two states for each subset of P(in the subset or not in the subset), it is therefore inferable that the number of subsets for P is $2^{|S|}$.

Problem set:

Based on the above information, write the answers to the following questions.

1. |{1,2,3,4,5,6,7,8,9,0}|

2. |P({1,2,3})|

3. P({0,1,2})

4. P({1})

Answers

1. 10
2. $2^3=8$
3. {{},{0},{1},{2},{0,1},{0,1,2},{0,2},{1,2}}
4. {{},{1}}

The Cartesian Product

The *Cartesian Product* of two sets is the set of all tuples made from elements of two sets. We write the Cartesian Product of two sets A and B as A × B. It is defined as:

$$A \times B = \{(a,b) | a \in A \text{ and } b \in B\}$$

It may be clearer to understand from examples;

$$\{0,1\} \times \{2,3\} = \{(0,2),(0,3),(1,2),(1,3)\}$$
$$\{a,b\} \times \{c,d\} = \{(a,c),(a,d),(b,c),(b,d)\}$$
$$\{0,1,2\} \times \{4,6\} = \{(0,4),(0,6),(1,4),(1,6),(2,4),(2,6)\}$$

It is clear that, the cardinality of the Cartesian product of two sets A and B is:

$$|A \times B| = |A||B|$$

A Cartesian Product of two sets A and B can be produced by making tuples of each element of A with each element of B; this can be visualized as a grid (which *Cartesian* implies) or table: if, *e.g.*, A = { 0, 1 } and B = { 2, 3 }, the grid is

×		**A**	
		0	**1**
B	**2**	(0,2)	(1,2)
	3	(0,3)	(1,3)

Problem set

Based on the above information, answer the following questions:

1. $\{2,3,4\}\times\{1,3,4\}$
2. $\{0,1\}\times\{0,1\}$
3. $|\{1,2,3\}\times\{0\}|$
4. $|\{1,1\}\times\{2,3,4\}|$

Answers

1. {(2,1),(2,3),(2,4),(3,1),(3,3),(3,4),(4,1),(4,3),(4,4)}
2. {(0,0),(0,1),(1,0),(1,1)}
3. 3
4. 6

Partitions of set:

Let S be a any nonempty set. A partition (∏) of S is a subdivision of S into nonoverlapping, nonempty subsets. A partition of S is a collection {Ai} of non-empty subsets of S such that:

1) $Ai \neq \emptyset$, where i=1,2,3,……
2) $Ai \cap Aj = \emptyset$ where $i \neq j$.
3) $UA_i = S$ where $A_1 \cup A_2 \cup .... \cup A_i = S$

Example 1:

let A = {1,2,3,n}
A1 = {1}, A2 = {3,n}, A3 = {2}
∏ = {A1, A2, A3} is a partition on A because it satisfy the three above conditions

Example 2 :

Consider the following collections of subsets of S = {1,2,3,4,5,6,7,8,9}

(i) [{1,3,5},{2,6},{4,8,9}]
(ii) [{1,3,5},{2,4,6,8},{5,7,9}]
(iii) [{1,3,5},{2,4,6,8},{7,9}]

Then

(i) is not a partition of S since 7 in S does not belong to any of the subsets.
(ii) is not a partition of S since {1,3,5} and {5,7,9} are not disjoint.
(iv) is a partition of S.

Mathematic induction:

It is useful for proving propositions that must be true for all integers or for a range of integer.
Proposition: is any statement P(n) which can be either true or false for each n in N. Suppose P has the following two properties.

Relations

There are many relations in mathematics :"less than" , "is parallel to ","is a subset of", etc. In a certain sense, these relations consider the existence or nonexistence of a certain connection between pairs of objects taken in a definite order. We define a relation simply in terms of ordered pairs of objects.

Product sets:
Consider two arbitrary sets **A** and **B** . The set of all ordered pairs (a ,b) where **a** $\in$ **A** and **b** $\in$ **B** is called the product, or cartesian product, of **A** and **B.**

$$A \times B = \{(a,b) : a \in A \text{ and } b \in B\}$$

Example:
Let $A = \{1,2\}$ and $B = \{a, b, c\}$ then
$A \times B = \{(1,a), (1,b),(1,c),(2,a),(2,b),(2,c)\}$
$B \times A = \{(a,1),(a,2),(b,1),(b,2),(c,1),(c,2)\}$

Relations
Let **A** and **B** be sets. **A** binary relation, **R**, from **A** to **B** is a subset of **A**×**B.** If $(x,y) \in$ **R**, we say that x is R-related to y and denote this by x**R**y

if $(x,y) \notin$ **R**, we write $x \not R\, y$ and say that x is not **R**-related to y .
if R is a relation from **A** to **A** ,i.e. **R** is a subset of **A** × **A**, then we say that **R** is a relation on **A.**

The **domain** of a relation **R** is the set of all first elements of the ordered pairs which belong to **R**, and the **range** of **R** is the set of second elements.

Example 1:
Let $A = \{1, 2, 3, 4\}$. Define a relation R on A by writing $(x, y) \in R$ if $x < y$. Then
$$R = \{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\}.$$

Example 2:
let $A = \{1,2,3\}$ and $R = \{(1,2),(1,3),(3,2)\}$. Then R is a relation on A since it is a subset of AxA with respect to this relation:
1R2, 1R3, 3R2 but $(1,1) \notin R$ & $(2,1) \notin R$

The domain of R is {1,3} and
the range of R is {2,3}

Example 3:

Let A = {1, 2, 3}. Define a relation R on A by writing (x, y) ∈ R , such that a≥b, list the element of R

$aRb \leftrightarrow a \geq b\ ,\ a,b \in A$

∴ R = {(1,1),(2,1), (2,2), (3,1), (3,2), (3,3)}.

Representation of relations:

1) By language
2) By ordered pairs
3) By arrow form
4) By matrix form
5) By coordinates
6) By graph form

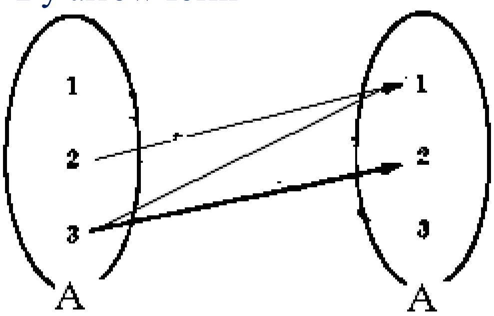
Example 4:

Let A = {1,2,3}, the relation R on A such that: $aRb \leftrightarrow a>b$; $a,b \in A$

1) By language:
   R={(a,b) : a,b ∈A and aRb ↔ a>b}

2) By ordered pairs
   R = {(2,1),(3,1),(3,2)}

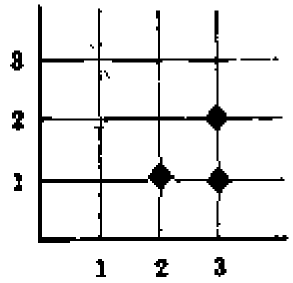
3) By arrow form



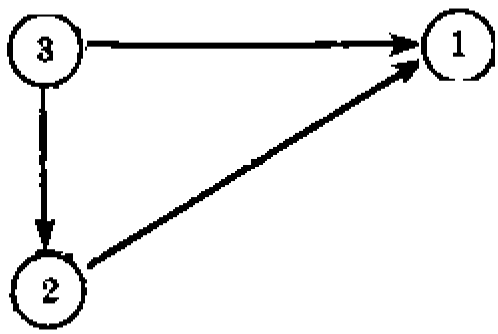
4) By matrix form

	1	2	3
1	0	0	0
2	1	0	0
3	1	1	0

5) By coordinates



6) By graph form



Properties of relations :

Let R be a relation on the set A

1) Reflexive : R is reflexive if : $\forall$ a $\in$A $\rightarrow$ aRa or (a,a) $\in$ R ; $\forall$ a, b $\in$A
2) Symmetric : aRb $\rightarrow$ bRa $\forall$ a,b $\in$A
3) Transitive : aRb $\wedge$ bRc $\rightarrow$ aRc
4) Equivalence relation : it is Reflexive & Symmetric & Transitive
5) Irreflexive : $\forall$ a $\in$A (a,a) $\notin$ R
6) AntiSymmetric : if aRb and bRa $\rightarrow$ a=b
   the relations $\geq,\leq$ and $\subseteq$ are antisymmetric

Example 5:

Consider the relation of C of set inclusion on any collection of sets:

1) A $\subset$ A for any set, so $\subset$is reflexive
2) A $\subset$ B dose not imply B $\subset$ A, so $\subset$ is not symmetric
3) If A $\subset$ B and B $\subset$ C then A $\subset$ C, so $\subset$ is transitive
4) $\subset$ is reflexive, not symmetric & transitive, so $\subset$ is not equivalence relations
5) A $\subset$ A, so $\subset$ is not Irreflexive
6) If A $\subset$ B and B $\subset$ A then A = B, so $\subset$ is anti-symmetric

Example 6:

If A ={1,2,3} and R={(1,1),(1,2),(2,1),(2,3)}

Is R equivalence relation ?

1) 2 is in A but (2,2) $\notin$ R, so R is not reflexive
2) (2,3) $\in$ R but (3,2) $\notin$ R, so R is not symmetric
3) (1,2) $\in$ R and (2,3) $\in$ R but (1,3) $\notin$ R, so R is not transitive

So R is not Equivalence relation

Example 7 :

What is the properties of the relation =?

1) a=a for any element a $\in$ A, so = is reflexive
2) If a = b then b = a, so = is symmetric
3) If a = b and b = c then a = c, so = is transitive
4) = is (reflexive + symmetric + transitive), so = is equivalence
5) a = a, so = is not Irreflexive
6) If a = b and b = a then a = b, so = is anti-symmetric

Inverse relations:

$$R^{-1} = \{(b,a) : (a,b) \in R\}$$

Example 1 :

Let R be the following relation on A ={1,2,3}

R = {(1,2),(1,3),(2,3)}

$\therefore$ R^{-1} = {(2,1),(3,1),(3,2)}

The matrix for R :

$$MR = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix},$$

and

$$MR\text{–}1 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix},$$

MR^{-1} is the transpose of matrix R

Composition of relations:

Let A, B, C be sets and let :

$R : A \rightarrow B$ $\quad (R \subset A \times B)$

$S : B \rightarrow C$ $\quad (S \subset B \times C)$

There is a relation from A to C denoted by

$R \circ S$ (composition of R and S)

$R \circ S : A \rightarrow C$

$R \circ S = \{(a,c) : \exists\, b \in B$ for which $(a,b) \in R$ and $(b,c) \in S\}$

Example :

let $A = \{1,2,3,4\}$

$B = \{a, b, c, d\}$

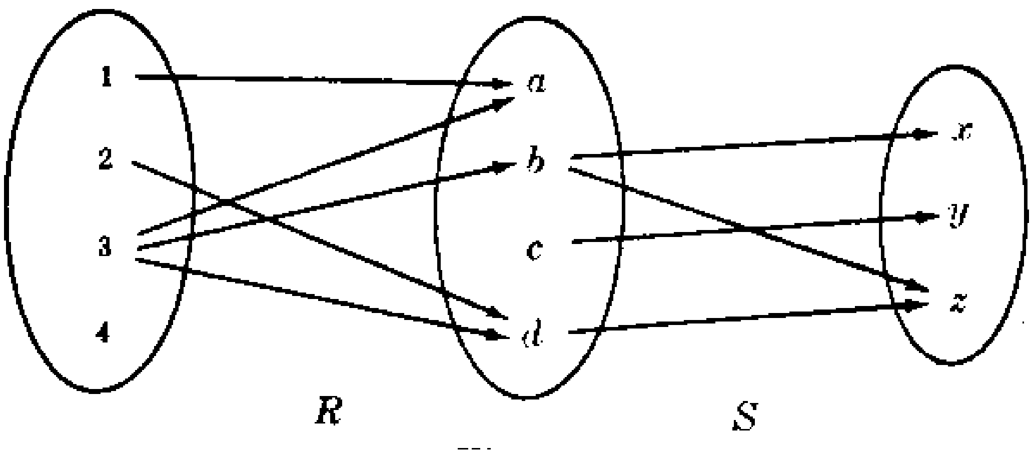
$C = \{x, y, z\}$

$R = \{(1,a),(2,d),(3,a),(3,d),(3,b)\}$

$S = \{(b,x),(b,z),(c,y),(d,z)\}$. Find $R \circ S$?

Solution :

1) The first way by arrow form



There is an arrow (path) from 2 to d which is followed by an arrow from d to z

2Rd and dSz $\Rightarrow$ $2(R \circ S)\,z$

$R \circ S = \{(3,x),(3,z),(2,z)\}$

2) The second way by matrix:

$$MR = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$MS = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

$R \circ S = M_R \cdot M_{S} =$

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

$R \circ S = \{(2,z),(3,x),(3,z)\}$

EXERCISE:

1- Let A={a,b} and B={4,5,6}

a. List the elements in A×B
b. List the elements B ×A
c. List the elements A×A
d. List the elements B×B

2- If A={a,b,c} B={1,2} and C={d,e} List all elements in A × B × C

3- If A $\subseteq$ C and B $\subseteq$ D, show that A × B $\subseteq$ C × D.

4- A=Z: a R b and only if a+b is even.

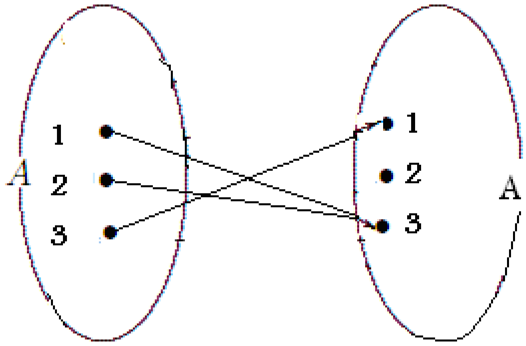
5- Consider the following arrays

VERT={1,2,6,4} , TAIL={1,2,2,4,4,3,4,1} , HEAD={2,2,3,3,4,4,1,3}, NEXT={8,3,0,5,0,0,0}, these describe a relation R on the set A={1,2,3,4}. Compute both the digraph of R and the matrix M_g .

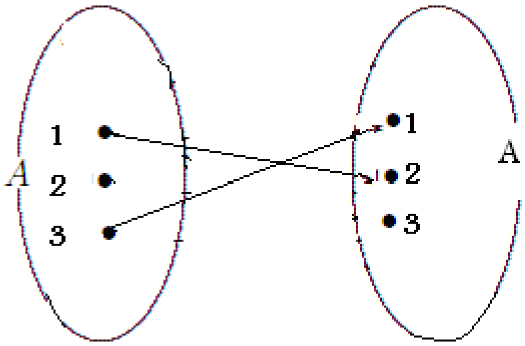
Function:

Function is an important class of relation.
Definition:
Let A,B be two nonempty sets, a function F: A→B is a rule which associates with **each** element of A a **unique** element in B.
The set A is called the **domain** of the function, and the set B is called the **range** of the function.

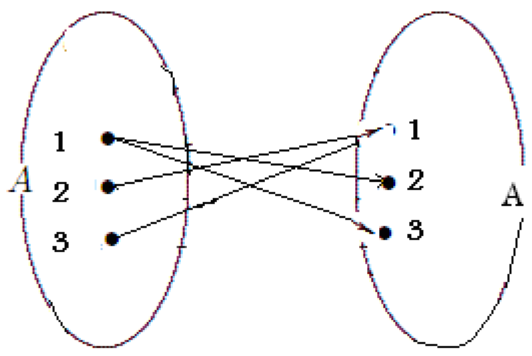
Example:
consider the following relation on the set A={1,2,3}
F = {(1,3),(2,3),(3,1)}
F is a function



G = {1,2},(3,1)}
G is not a function from A to A



H = {(1,٣),(2,1),(1,2),(3,1)}
H is not a function

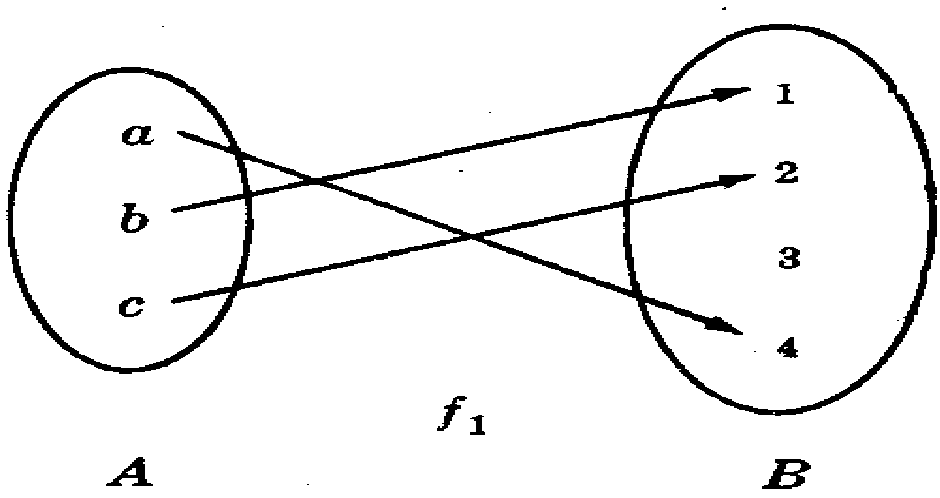


One-to-one ,onto and invertible functions :

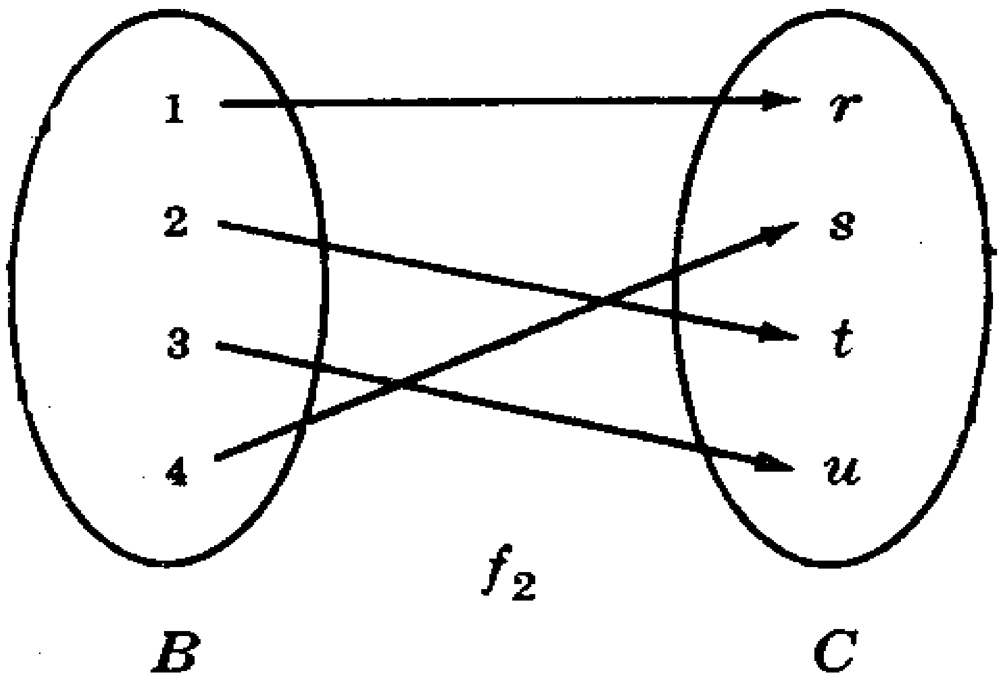
1) One –to-one : a function $F:A \rightarrow B$ is said to be one-to-one if different elements in the domain (A) have distinct images.
   If $F(a) = F(a') \Rightarrow a = a'$
   $F:A \rightarrow B$ is one-to-one if different elements in A have distinct images

2) Onto
   $F:A \rightarrow B$ is said to be an onto function if each element of B is the image of some element of A.
   $\forall\ b \in B \quad \exists \quad a \in A : F(a) = b$

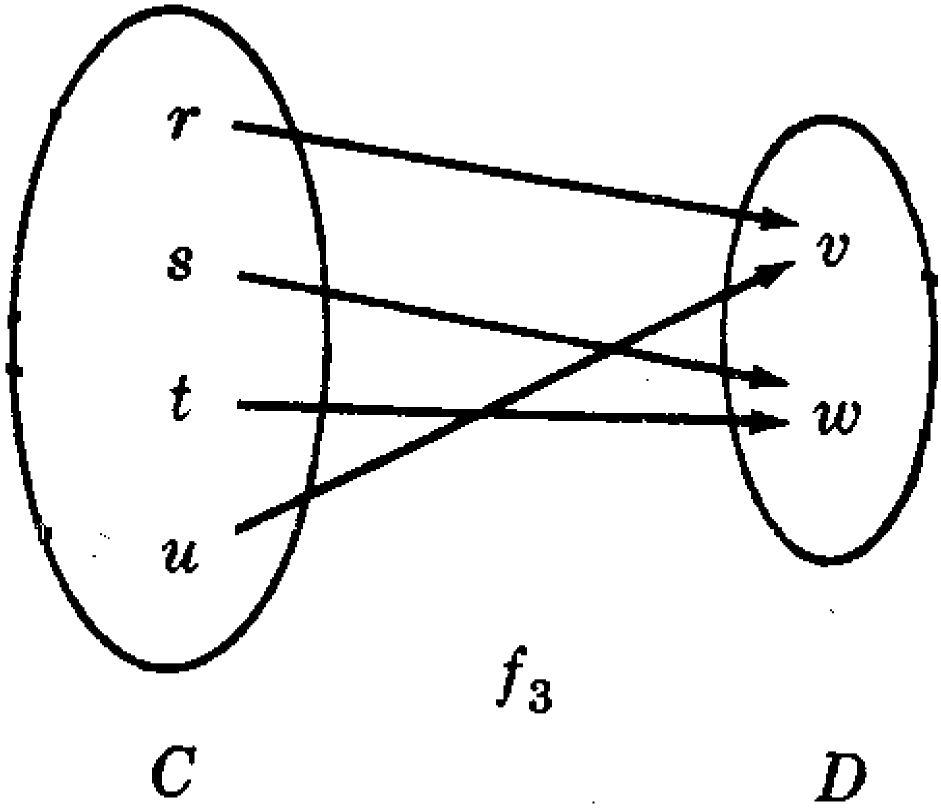
3) Invertible (One-to-one correspondence)
   $F:A \rightarrow B$ is invertible if its inverse relation f^{-1} is a function $F:A \rightarrow B$
   $F:A \rightarrow B$ is invertible if and only if F is **both** one-to-one and onto
   $F^{-1} : \{(b,a)\ \forall\ (a,b) \in F\}$



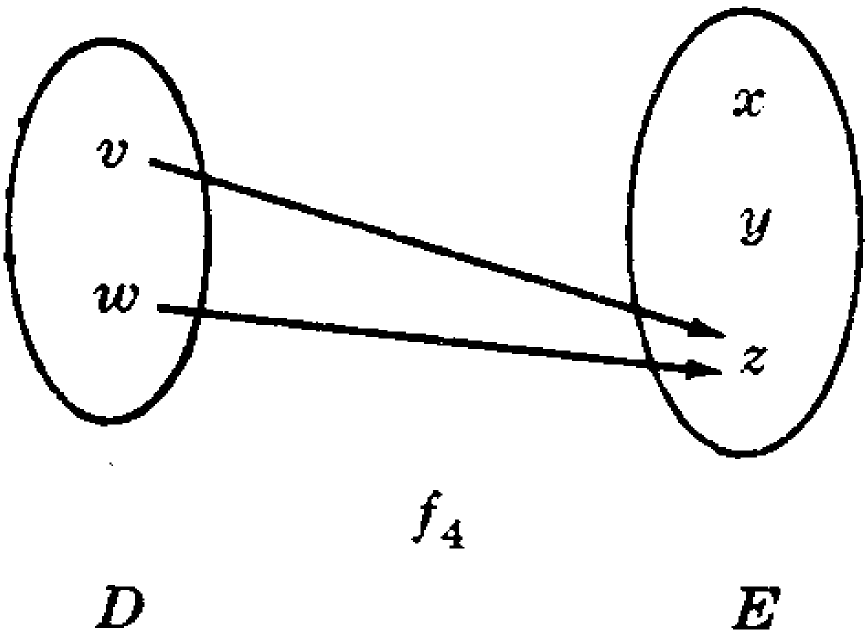
one to one & not onto [3∈B but it is not the image under f1]



both one to one & onto
(or one to one correspondence between A and B)



not one to one & onto



not one to one & not onto

We can also describe the same information in the following transition table:

	ν	
	0	1
$+s_1+$	–	s_2
s_2	s_3	s_4
s_3	–	s_2
s_4	s_5	s_7
s_5	s_6	s_9
s_6	–	s_4
s_7	–	s_8
$-s_8-$	–	–
$-s_9-$	–	–

EXERCISE:

1-Consider a deterministic finite automata which will recognize the input string 1(01) *and 1(11)*(0+1) ,and nothing else, then decide which of the following string are accepted by this automata:
111,1111,11111,10110,1101,101110.